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INTRODUCTION

For the training of specialists in the construction industry, graphic education is essential. The study of descriptive geometry as a theory of constructing projection images on a drawing, engineering and computer graphics allows you to acquire the knowledge and skills necessary to complete and read construction drawings. By studying graphic disciplines, students develop their spatial imagination. Without it, it is impossible to design, create models using graphic programs, or read technical drawings. Information on the theory of drawing definition and solving projection problems (supplemented by knowledge of the requirements for the design of drawings) is absolutely necessary when developing projects for architectural and construction purposes.

The materials of the teaching aid are an addition to practical exercises and provide independent solid assimilation of educational material in a limited time. The content and sequence of the arrangement of the material are subordinated to the task of forming a student's deep and integral understanding of the projection method and methods of developing design documentation, as well as working with it.

This manual also contributes to more productive independent work of students when doing homework. The manual briefly outlines the theoretical provisions on the basis of which the solution of problems is carried out, the procedure for solving them is considered in detail, and the necessary methodological instructions are given. A significant part of the theoretical section is devoted to methods for solving problems on the topic “Surfaces”.

The educational and methodical manual is compiled in accordance with the curricula for teaching the main construction specialties of higher educational institutions; it can also be used by students of all areas of study of full-time and part-time departments.

1. PROJECTION DRAWING THEORY

Not every image of an object on a sheet of paper allows you to accurately determine its geometric shape. Therefore, it is necessary that the image of an object on a plane be built according to certain geometric rules that allow one to move from flat forms to the spatial forms of the depicted object. Such a geometrically regular image of an object on a plane is achieved using the projection method, which is the basis of descriptive geometry. Descriptive geometry studies methods for depicting spatial forms on a plane and methods for solving geometric problems using drawings.

This chapter of the manual contains solutions to basic problems in the course of descriptive geometry. For a better study of the material, it is also necessary to carefully read the relevant sections of the textbooks.

1.1. Basic positional and metric tasks on the complex drawing

The construction of an image of any spatial form on a plane is reduced to finding its projection onto this plane, i.e. construction of a projection drawing. Before studying the methods of transition from a spatial model to a projection drawing, we will consider sequentially the construction of the projection image of a point, a line and a plane.

1.1.1. Fundamentals of projection image on drawings. Point, line, plane

Projection of points by coordinates on the orthogonal drawing

To build projections of any point located in space, you must specify its coordinates (X, Y, Z). Let's set some point and consider its construction on the Monge diagram.

Let's choose the following positive directions of the projection axes (fig. 1.1): the X-axis is to the left, the Y-axis is down, the Z-axis is up.

To construct projections of a point, it is necessary to set aside the value of the X coordinate from the origin along the $0X$ axis and draw a straight line perpendicular to this axis from the resulting point. Having set aside the value of the Y coordinate of a given point on this straight line from the X axis, we will get its horizontal projection on the diagram. To obtain a frontal projection of a point, it is necessary to set aside the value of the Z coordinate of this point on this straight line.

When settling coordinates, their sign should be taken into account. For example (see fig. 1.1), point C has a negative sign Y coordinate, so its value is plotted upwards from the $0X$ axis.

The horizontal bcd and frontal $b'c'd'$ projections of the triangle BCD will be obtained by connecting the projections of the same names of points B, C and D with straight lines.

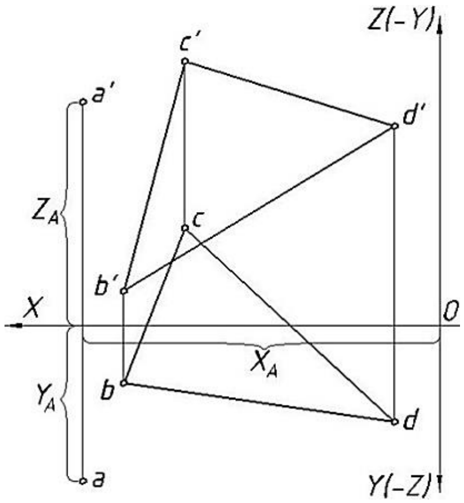


Fig. 1.1. Construction of projection points $B(b, b')$, $C(c, c')$, $D(d, d')$ on the Monge diagram

Definition of plane traces and determination of its visibility

A straight line belonging to the plane and not parallel to any projection plane intersects the projection plane at a point. This one lies on the line of intersection of the given plane with the plane of projections (trace of the plane). Therefore, to construct traces of a plane, it suffices to construct the intersection points of two arbitrary lines lying in the plane with projection planes (traces of these lines). When connecting traces of the same name with each other by straight lines, we obtain the corresponding traces of a given plane. The point of their intersection (vanishing point of traces) must lie on the X axis.

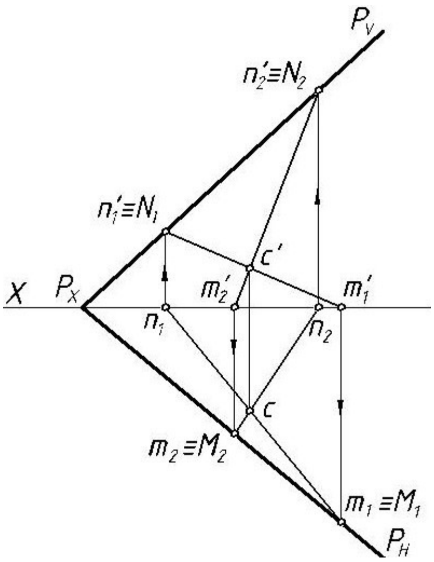


Fig. 1.2. Construction of plane *P* traces

To determine the horizontal trace of a straight line (fig. 1.2), it is necessary to draw a perpendicular to the axis from the point of intersection of the straight line frontal projection with the *X* axis until it intersects with the horizontal projection of this straight line. We obtain the horizontal trace M_1 (M_2) of the line and its horizontal projection m_1 (m_2).

To determine the frontal trace of a straight line (see fig. 1.2), it follows from the point of intersection of its horizontal projection with the *X* axis to draw a perpendicular to the axis until it intersects with the frontal projection of the straight line. The point of intersection will be the frontal trace of the straight line N_1 (N_2) and its frontal projection n'_1 (n'_2).

On fig. 1.3, in order to construct a horizontal trace P_H of the plane *P*, the horizontal traces of the lines *BC* (*bc*, *b′c′*) and *CD* (*cd*, *c′d′*), points M_1 and M_2, are constructed using the method discussed above.

The frontal trace of the straight line *CD* (*cd*, *c′d′*) (point N_1) and the vanishing point of the traces P_X have to be found in order to construct the frontal trace P_V of the plane.

On fig. 1.4 there is triangle *BCD* (*bcd*, *b′c′d′*), located in the first and second quarters of space. To construct traces of the given plane, we construct traces of its sides *DC* (*dc*, *d′c′*) and *BC* (*bc*, *b′c′*). Firstly, the point n_2 of intersection is found in the horizontal projection of the line *DC* (*dc*, *d′c′*) with the axis *X* (which is the horizontal projection of the frontal trace of this line). We find the frontal projection n'_2 and the frontal trace N_2 itself at the intersection of the perpendicular drawn to the axis *X* from the point n_2 with the frontal projection of the straight line *DC* (*dc*, *d′c′*). Similarly, we find the projections n_1 and n'_1 of the frontal trace and the frontal trace N_1 of the line *BC* (*bc*, *b′c′*) itself.

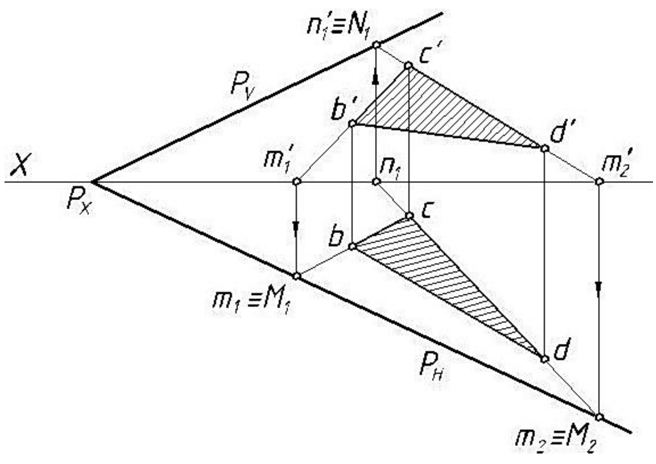


Fig. 1.3. Construction of BCD plane traces

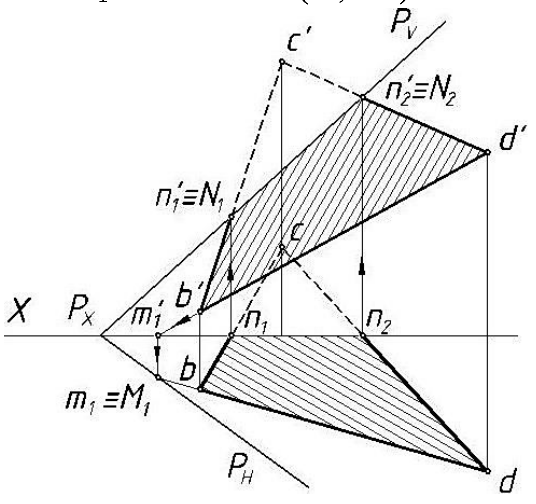


Fig. 1.4. Determining the visibility of triangle *BCD*

The frontal trace P_V of the plane *BCD* (*bcd*, *b′c′d′*) is obtained by connecting the points N_1 and N_2, the frontal traces of the lines *BC* (*bc*, *b′c′*) and *DC* (*dc*, *d′c′*), with straight lines.

Let us construct a horizontal trace of the straight line *BD* (*bd*, *b′d′*). To do this, we mark the point of intersection of the frontal projection of this straight line with the *X* axis (point m'_1) — the frontal projection of the horizontal trace of the straight line. From this point we draw a perpendicular to the *X* axis until it intersects with the horizontal projection of the straight line *BD* (*bd*, *b′d′*) at the point m_1. The resulting point is the horizontal trace of this line M_1. Connecting the point M_1 with the vanishing point of traces P_X constructed earlier, we obtain the desired horizontal trace P_H of the plane *BCD* (*bcd*, *b′c′d′*).

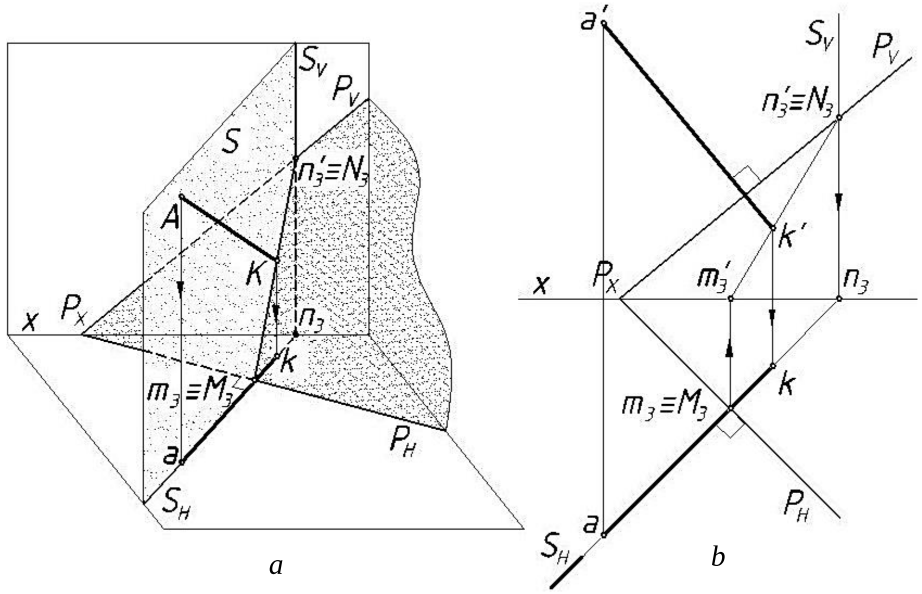
The front track of the P_V plane cuts off those parts of the triangle *BCD* (*bcd*, *b′c′d′*) that are outside the first quarter of space. In fig. 1.4, the part of the triangle adjacent to the vertices *B* (*b*, *b′*) and *D* (*d*, *d′*) in the first quarter is visible. Projections of the visible part of the triangle are hatched. The part of the triangle with a vertex *C* (*c*, *c′*) that is behind P_V goes into the second quarter of space and is invisible. The projections of this part are shown by the invisible outline line.

Definition of distance from point to plane

It is known that the shortest distance from a point to a plane is equal to the length of a perpendicular lowered from a point to a plane. Therefore, to solve the problem on the orthogonal drawing, it is necessary to draw a line through the point perpendicular to the given plane. Then construct the intersection point of the conducted perpendicular with the plane and determine the real size of the segment between the given point and the intersection point. Let's consider a step-by-step solution of this problem.

Drawing a line perpendicular to a plane through a point. If the line is perpendicular to the plane, then its projections are perpendicular to the traces of the same plane or corresponding projections of the horizon and front of the plane.

For example, in fig. 1.5, the horizontal projection of the perpendicular (HPP) lowered from the point *A* to the plane *P* (P_V, P_H) is perpendicular to its horizontal trace P_H, and the frontal projection of the perpendicular (FPP) is respectively perpendicular to P_V.



a *b*

Fig. 1.5. Drawing a perpendicular to the plane *P* (P_V, P_H), given by the following traces:
a — in axonometry; *b* — on Monge diagram

On fig. 1.6 the plane is given by the triangle *BCD*. In order to draw a perpendicular to a given plane, we build a horizontal and a frontal in the plane. Then, through the projection of the point *a′*, we draw the frontal projection of the perpendicular at a right angle to the frontal projection of the frontal (FPF), and through the projection of the point *a* — the horizontal projection of the perpendicular at a right angle to the horizontal projection of the horizontal (HPH). The second step in determining the required distance is to construct a point of intersection with the plane *BCD*.

Construction of the intersection point of a perpendicular with a plane. To solve the problem of constructing the intersection point of a perpendicular with a plane, it is necessary to:

– draw an auxiliary projecting plane through the perpendicular;

– to construct a line of intersection of the given and auxiliary planes;

– it is necessary to determine at the intersection point of the constructed line with a perpendicular the desired intersection point of the perpendicular with the plane.

On fig. 1.5 the plane P (P_V, P_H), to which the perpendicular is drawn from the point A, is given by traces. The projections of the perpendicular form a right angle with the corresponding traces of the plane. To determine the point of intersection of the perpendicular with the plane P (P_V, P_H), we draw a horizontally projecting plane S through it (the horizontal trace S_H of this plane passes along the horizontal projection of the perpendicular).

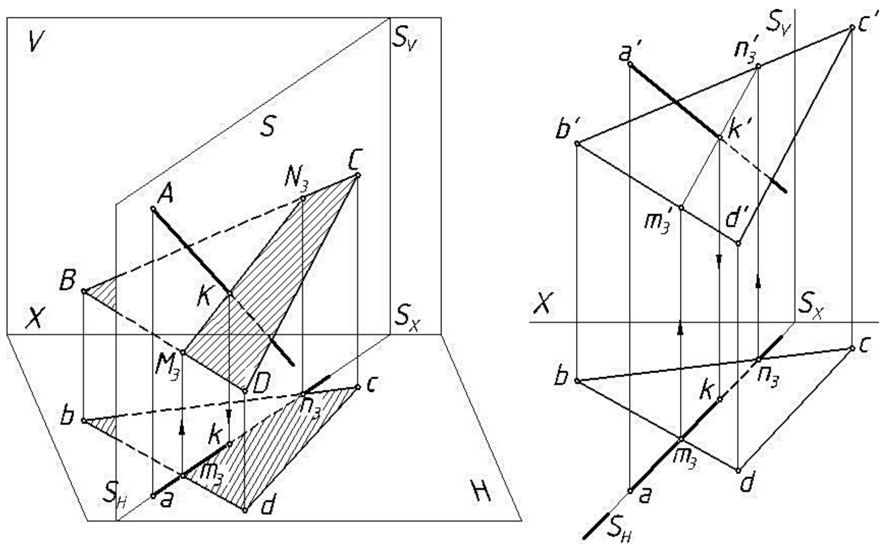
By connecting the intersection points of the same-name traces of the planes P and S (M_3 and N_3), we obtain the line of intersection of these planes.

Having constructed the projections of these points and connecting their similar projections, we obtain the projections of the line of intersection corresponding to the given plane P and the auxiliary plane S (lines m_3n_3 и $m'_3n'_3$).

The k' intersection point of the frontal projections of the perpendicular and the intersection line of the planes is a frontal projection of the intersection point of the perpendicular with the *BCD* plane.

The horizontal projection of the intersection point k is located along the projection connection on the horizontal projection of the perpendicular.

On fig. 1.6 the plane is given by the triangle *BCD*. Following the plan for solving the problem, we draw a horizontally projecting plane S through the straight line. Its horizontal trace S_H coincides with the projection of the perpendicular, the frontal trace S_V is perpendicular to the projection axis X.



a *b*

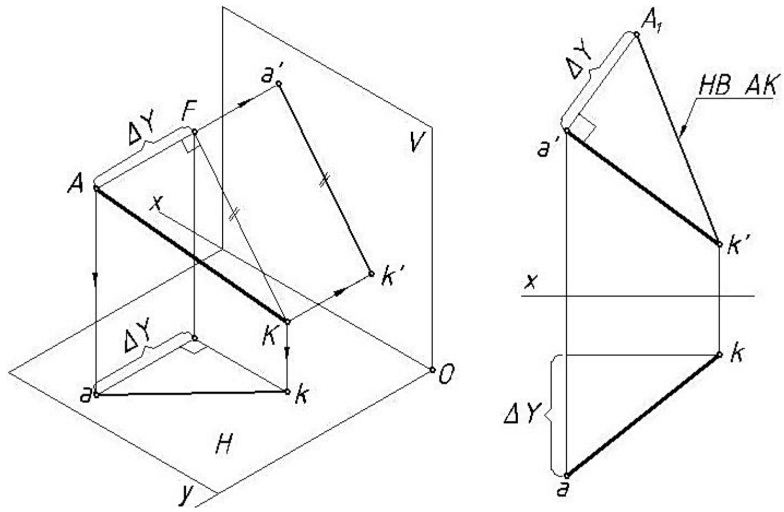
Fig. 1.6. Drawing a perpendicular to the plane given by triangle *BCD*:
a — in axonometry; *b* — on Monge diagram

Then we determine the horizontal projection (m_3n_3) of the line of intersection of the planes *BCD* and S. Its frontal projection ($m'_3n'_3$) is found from the condition that the points M_3 and N_3 belong to the sides of the triangle *BD* and *BC*, respectively.

At the intersection $m'_3n'_3$ with the frontal projection of the perpendicular, we mark the frontal projection k' of the desired point K (k, k'). The horizontal projection of the intersection point k is located along the line of communication on the horizontal projection of the perpendicular.

Definition of the perpendicular's natural size value. Determination of the natural value of the perpendicular. The natural value of the segment *AK* (fig. 1.7) is determined by the method of a right triangle. One of the legs of such a triangle is the projection of the segment, and the second is the difference in the coordinates of the end points of the segment, taken on the second projection plane. The natural size of the segment is equal to the hypotenuse of this triangle.

On fig. 1.7 triangle *KFA* is drawn; the angle at the vertex F is straight, and the leg *KF* is equal in length to the frontal projection of segment *AK*. The second leg of this triangle — *AF* is equal to the difference of the $0Y$ coordinates of the end points of the segment *AK*.



a *b*

Fig. 1.7. Definition of the natural size value of the perpendicular segment *AK*:
a — in axonometry; *b* — on Monge diagram

To construct the natural size of the segment *AK* on an orthogonal drawing, it is necessary to set aside the difference in the coordinates of the ends of the segment Δ*Y* at right angles to the frontal projection of the segment *AK*. The hypotenuse of the resulting triangle (segment $k'A_1$) is the desired distance from point *A* to the plane *BCD* (*bcd*, *b′c′d′*).

Drawing a plane parallel to a given plane and separated from it at a certain distance

To solve the problem, it is necessary to initially construct a point that is separated from the plane at a given distance and then to pass through it a plane parallel to the given plane. Consider solving the problem step by step.

Construction of a point separated from the plane on a given distance. We take the specified distance equal to 30 mm. We choose an arbitrary point in the plane *P* (fig. 1.8). To do this, we draw a horizontal line in the plane, on which we take the point *K* (*k*, *k′*). From the chosen point *K* (*k*, *k′*), we restore the perpendicular *KA* (*ka*, *k′a′*) of arbitrary length to the plane *P* (P_V, P_H).

To determine the point we need on the perpendicular *KA* (*ka*, *k′a′*), which is 30 mm away from the plane, we construct the actual size of the perpendicular *KA* (*ka*, *k′a′*). Putting aside the given 30 mm from the point *k′*, we get the point T_1, and dropping the perpendicular from this point to the frontal projection of the segment *a′k′*, we get the point *t′*. It is a frontal projection of a point 30 mm away from the *P* plane. The horizontal projection of the point t is determined in the projection relation on the horizontal projection of the perpendicular *ak*.

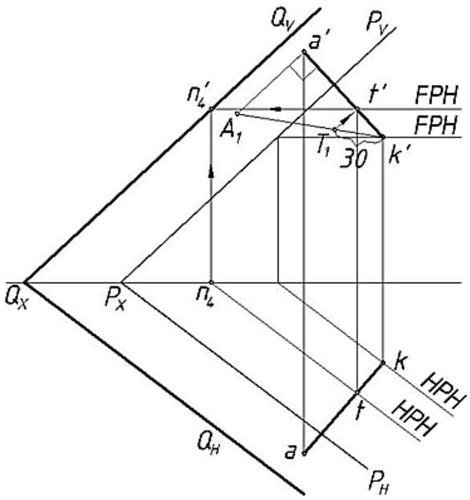


Fig. 1.8. Drawing the plane *Q* parallel to the given plane *P* at a distance of 30 mm

Constructing a plane passing through a given point, parallel to a given plane. Now through the constructed point it is necessary to draw a plane parallel to the given plane. To do this, we first draw a straight line parallel to this plane through a point. Having constructed traces of a straight line, we can draw traces of the desired plane through them, parallel to the traces of a given plane.

On fig. 1.8 a horizontal line is drawn through the point *T* (*t*, *t′*). Its frontal projection (FPH) passes through the frontal projection of the point *t′* parallel to the 0*X* axis, and the horizontal projection (HPH) passes through the horizontal projection of the point *t*, parallel to the horizontal trace of the plane.

Having determined the frontal trace of the drawn horizontal at the point n'_4, we draw through it the frontal trace of the desired plane Q (Q_V, Q_H), parallel to the frontal trace of the plane P (P_V, P_H).

At the intersection point of the constructed trace with the $0X$ axis, we mark the vanishing point of the traces, and draw a horizontal trace of the plane through this point, parallel to the horizontal trace of the plane P (P_V, P_H).

The constructed plane Q (Q_V, Q_H) satisfies the exiting condition, since it is parallel to the given plane P (P_V, P_H) (the traces of the same name are parallel) and passes through the point T (T_V, T_H), which is 30 mm away from the plane P (P_V, P_H) (because it contains a straight line passing through the point T (T_V, T_H)).

Construction of a plane passing through an arbitrary point and perpendicular to a line

Let's consider the case when the desired plane must be constructed perpendicular to an arbitrary straight line and at the same time pass through an arbitrary point in space.

From the condition presented in the drawing (fig. 1.9), the traces of the desired plane must be perpendicular to the same-named projections of the straight line CD (cd, $c'd'$), while the plane itself must pass through the point E.

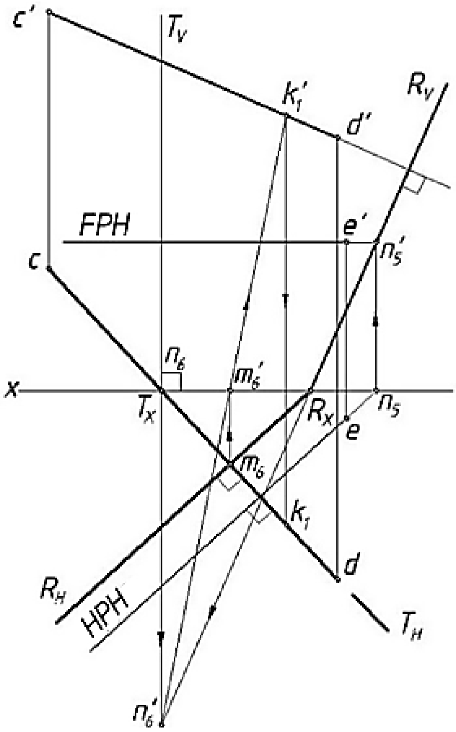


Fig. 1.9. Definition of a plane R (R_V, R_H) passing through point E (e, e'), perpendicular to a given line CD (cd, $c'd'$)

To solve the problem through the point E, we can draw the main straight line of the desired plane, for example, a horizontal line. We know the directions of its projections: the horizontal projection of the horizontal (HPH) should be perpendicular to the projection of the same name of the straight line CD (cd, $c'd'$), and the frontal projection (FPH) should be parallel to the X axis. Having found the frontal trace (see fig. 1.9) of the drawn horizontal (point n'_5) we obtain a point through which the frontal trace of the desired plane R (R_V, R_H) must pass, perpendicular to the frontal projection of the straight line CD — $c'd'$ (R_V). This trace is prolonged until it intersects with the X-axis.

We get the vanishing point of traces R_X, through which, perpendicular to the horizontal projection of the straight line CD (cd, $c'd'$), we draw a horizontal trace of the plane R_H.

Definition of an intersection point constructed by a plane and a line. The construction of the point of intersection of the line CD (cd, $c'd'$) with the constructed plane R (R_V, R_H) is carried out in the following steps:

1. An auxiliary projecting plane T (T_V, T_H) is drawn through the straight line CD (cd, $c'd'$). Its horizontal track T_H coincides with the horizontal projection of the straight line (cd), and the frontal T_V is perpendicular to the X axis.

2. The line of intersection of the auxiliary plane T with the previously constructed plane R (R_V, R_H) is determined — a straight line (m_6n_6, $m'_6n'_6$).

3. The point of intersection of the side CD (cd, $c'd'$) with the constructed plane is determined by its projections (k_1, k'_1).

1.2. Surface projection

When constructing drawings it is often necessary to solve so-called positional problems, i.e. to determine the belonging of one geometric element to another or solve the problems on their mutual intersection. To do this, you have to construct a point on the plane or on the surface. So before we get to the solution of the problems, let's look at the projections of the basic geometric solids, which may consist of parts, on the plane of projections.

1.2.1. Definition of geometric body projections

Prism projections

Fig. 1.10 shows the projection of a straight quadrilateral prism.

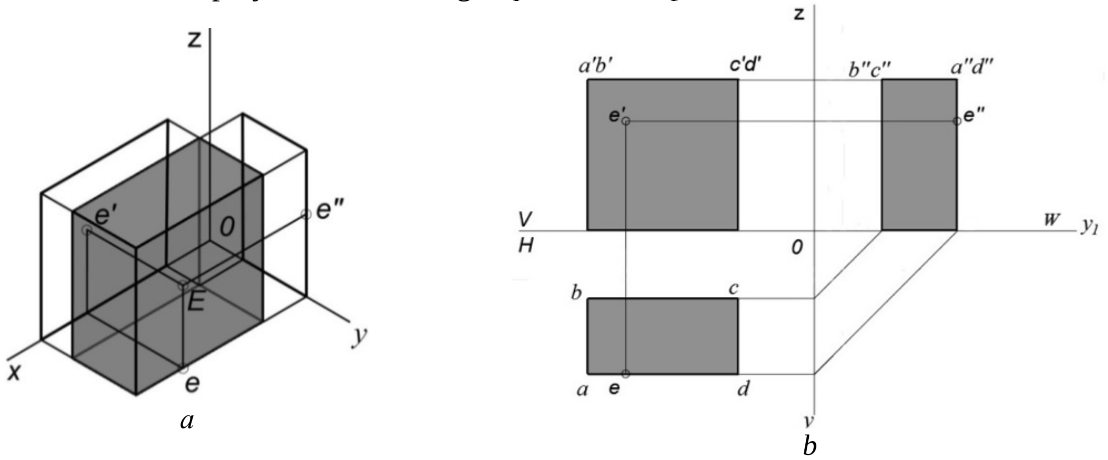


Fig. 1.10. Constructing a point on the surface of a straight prism:
a — in axonometric view; *b* — in three projections on the Monge diagram

The projections of points lying in the face plane are defined through lines, which are projected to the horizontal plane by points lying on the base side. Figure 1.10 shows the construction of points *E* (*e*, *e′*, *e′′*) lying on the plane of the front faces.

Pyramid projections

Fig. 1.11 shows pyramid projections.

Frontal and profile projections are constructed on a given horizontal projection of a point lying on the plane of the pyramid's face (see fig. 1.11).

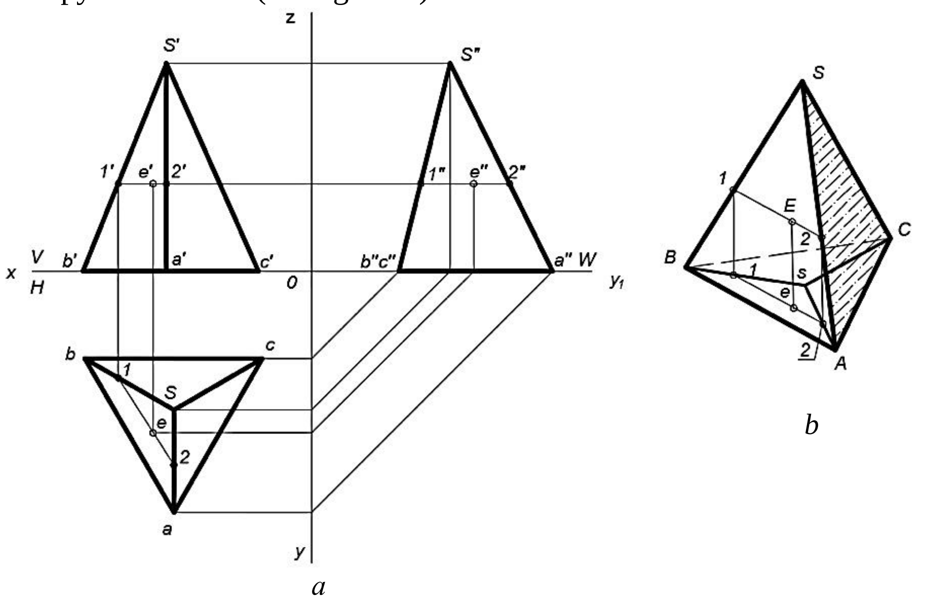


Fig. 1.11. Construction of projections of a point on the surface of the pyramid:
a — in three projections on the Monge diagram; *b* — in axonometric view

According to the given horizontal projection *e* of the point *E*, lying on the face plane *ASB* of the pyramid, the frontal *e′* and profile *e′′* projections are constucted (see fig. 1.11).

Draw a straight line 1–2 through the point *e* parallel to the side of the base *ab*. The line 1–2 must belong to the projection of the face *ASB* (two points of this line lie on the edges of the face). We build a frontal projection 1′2′ parallel to *a′b′*. We determine the frontal projection *e′* along the line of the projection connection. Similarly, we find the profile projection *e′′*.

Cylinder projections

Fig. 1.12 shows projections of a right circular cylinder.

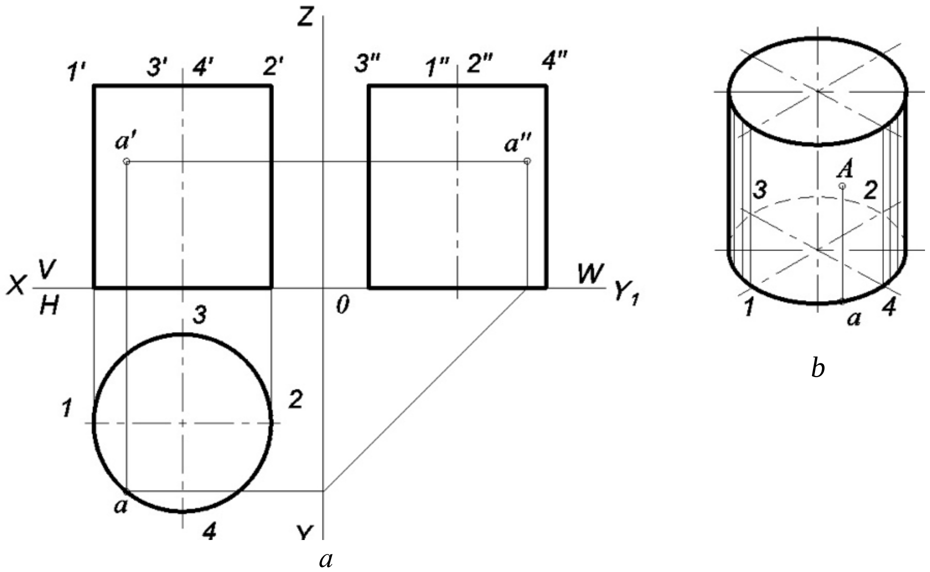


Fig. 1.12. Projection of points on the surface of the right circular cylinder:
a — in three projections on the Monge diagram; *b* — in axonometric view

On a horizontal plane, the entire lateral surface of the cylinder is projected into a circle. All generators, as projecting lines, are projected to points lying on the circle. The horizontal projection (*a*) of a point belonging to the surface of the cylinder coincides with the projection of the generatrix on which it lies. To build a profile projection, we find the position of the generatrix and along the line of the projection connection the projection *a*″ (see fig. 1.12).

Cone projections

Figure 1.13 shows the projections of a right circular cone.

The missing projections (see fig. 1.13). need to be constructed according to the given projections of points *A* and *B* which belong to the surface of the cone.

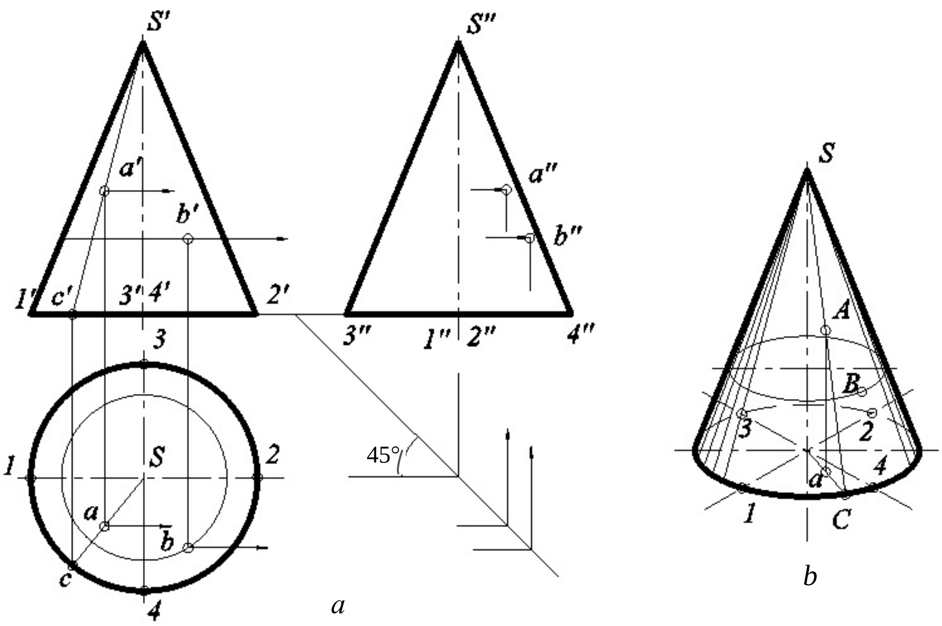


Fig. 1.13. Projection of points on the surface of a straight circular cone:
a — in three projections on the Monge diagram; *b* — in axonometric view

To construct the missing projections of points, it is necessary to draw a generatrix or parallel (circle) on the surface of the cone through a given projection and construct another projection along the line of the projection connection.

The construction of the projections of the point *A*(*a*, *a*′) is made using the generatrix *SC*, and the point *B*(*b*, *b*′) is made using the parallel. If the given point is located on the contour outline generatrix, then the projection of the point is found without drawing additional lines.

Sphere projections

A sphere is a surface formed by rotating a circle around its diameter. A sphere is projected onto all projection planes by a circle with a radius equal to the radius of the sphere. The projections of the sphere are shown in fig. 1.14.

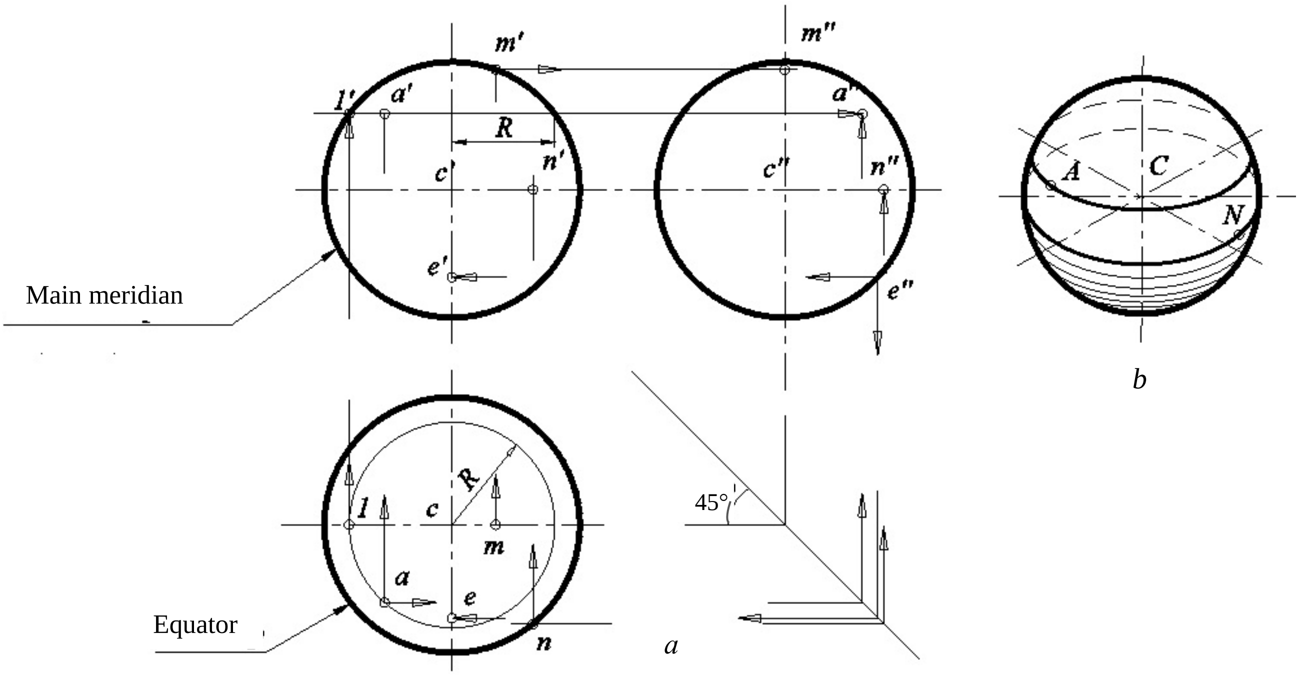


Fig. 1.14. Projection of a point on a sphere:
a — in three projections on the Monge diagram; *b* — in axonometric view

The contour line of the frontal projection of the sphere is the frontal projection of the main meridian, and the contour line of the horizontal projection of the sphere is the equator.

Construction of the missing projections of points on the surface of the sphere.

Fig. 1.14 shows the construction of point *A*(*a*, *a*′), lying on the surface of the sphere. To construct the missing projections, a parallel is drawn through the given projection of the point, and another projection of the point is found along the line of the projection connection. Point *M*(*m*, *m*′), which lies on the frontal outline, point *N*(*n*, *n*′), which lies on the horizontal outline, and point *E*(*e*, *e*′), which lies on the profile outline, are built without auxiliary lines.

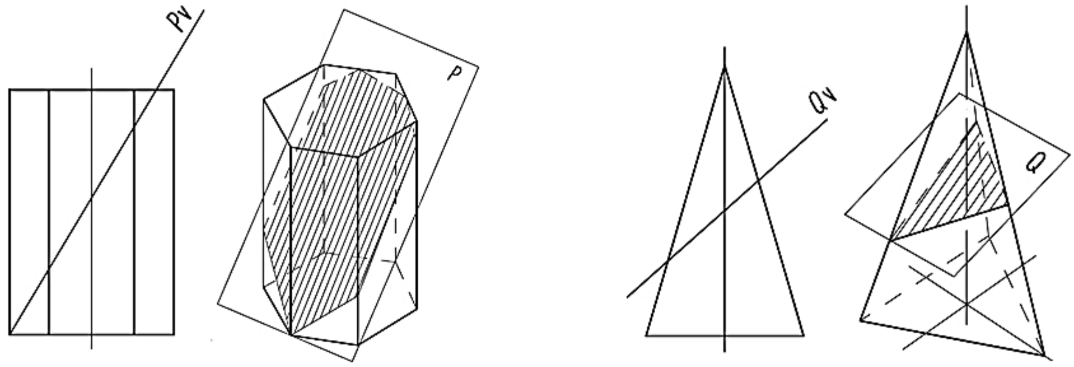
After studying the construction of geometric body projections, let’s move to the solution of problems related to the construction of the projection of details.

1.2.2. Sections of polyhedra and bodies of revolution by a plane

The proposed variants of the tasks are a combination of the most common geometric bodies in practice: a prism, a cylinder, a pyramid, a cone, a sphere, and secant planes intersecting all of them.

As a result of the surface’s intersection with a plane, a flat figure (section) is obtained, the shape of which depends on the type of surface and the position in space of the secant plane.

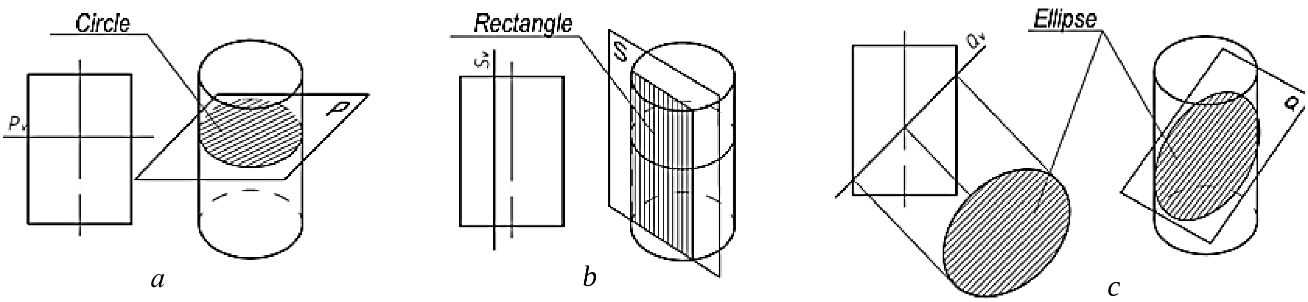
Thus, when intersecting a polyhedron with a plane, a polygon is obtained whose vertices are the intersection points of the polyhedron's edges with the secant plane (fig. 1.15).



a *b*

Fig. 1.15. Sections of a pyramid and a prism by a plane:
a — section of the prism; *b* — section of the pyramid

Depending on the location of the cutting planes relative to the axis of the cylinder in fig. 1.16 shows the resulting sections — a circle, a rectangle, an ellipse.



a *b* *c*

Fig. 1.16. Sections of the cylinder: *a* — circle; *b* — rectangle; *c* — ellipse

Various sections of a right circular cone depending on the location of the secant planes:

1) if the cutting plane is located with respect to the axis of the cone at an angle of 90°, then a circle is obtained in the section; if the cutting plane passes through the vertex of the cone, then the section is a triangle;

2) if the secant plane has an inclination angle α — less than the angle β of generating cone's inclination to the horizontal plane (the secant plane intersects all the generating planes), an ellipse is obtained in the section;

3) if $\alpha = \beta$ (the cutting plane is parallel to one of the cone's generatrix), then the section is a parabola;

4) if $\alpha > \beta \leq 90°$ (the cutting plane is parallel to two generators), then the section is a hyperbola.

Intersection of a surface with a plane of partial position

The intersection of the pyramid with the projecting plane. If the secan plane is a plane of particular position, then the construction of the line of intersection with the surface is greatly simplified, since one projection of this line becomes known and coincides with the trace of the secant plane. The problem is reduced to determining the missing projections of the points of the intersection line, which in this case are found as points lying on the given surface.

On fig. 1.17 to determine the horizontal and profile projections of the pyramid section, the points of intersection of the frontal projections of the ribs with the frontal trace of the cutting plane are marked. The horizontal and profile projections of these points are found on the corresponding projections of the edges using connection lines and are connected in a certain order.